Valid and Reliable Detection of Gerrymanders

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Abstract

Allegations of partisan gerrymandering are frequently heard but seldom persuasively confirmed. We diagnose the identification problem as stemming from presuming that gerrymanders can be detected by focusing on underserved seat wins and losses. Because gerrymanders come in two forms, cracking and packing, and because the forms produce dissimilar effects, and because a seat focus analyzes the effects jointly rather than separately, a focus on winning seats sows the seeds for doubts. We argue that a switch to a focus on two qualities of the vote percentage distribution among districts sidelines those doubts. We follow through on the argument through analyses of a possible cracking gerrymander of Massachusetts' 2012 congressional districts and a possible packing gerrymander of Ohio's 2012 congressional districts. Both analyses show the vote-distribution approach provides valid and reliable readings of gerrymanders, while various seat-focused approaches prove unreliable.

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1. Introduction

Partisan gerrymandering has operated as a form of electoral manipulation in American politics for centuries, pre-dating even the 1812 cartoon that gave the practice its name.[1] It is dismaying that after more than two centuries it remains an open question whether social scientists, and others for that matter, can validly and reliably identify a gerrymander. Allegations of partisan gerrymandering abound, but convincing evidence has been in short supply.[2] Our purposes are twofold: to diagnose why it has been so difficult to arrive at an agreed upon, valid, and reliable analysis of gerrymanders, and to offer methods for identifying gerrymanders that are demonstrably valid and reliable.

We contend that the persistent difficulties in persuasively identifying gerrymanders grow out of the near-universal focus on whether a set of districts produces excess seats for one party and a deficient number for the opposition. This focus on seats ignores the dissimilar effects that cracking versus packing gerrymandering can produce. Politically motivated mapmakers might crack one partisan group across multiple districts in a manner that relegates them to a minority in each district, denying members of the effected party the ability to win a seat. Alternatively, mapmakers might draw electoral districts in ways that pack partisans into a small number of districts, ensuring that members of one party will

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[1] For a recounting of the when, where, and how gerrymandering got its name, see Martis (2008). See Griffith (1907) and Engstrom (2013, 21-42) for discussions of gerrymandering in the United States prior to 1812.

[2] Even so infamous a set of line-drawing maneuvers as Phil Burton's congressional districting plans in California during the 1980s saw sharp disagreement between two of political science's distinguished districting authorities (Grofman 1985; Cain 1985). More recently the Supreme Court decided, with some reluctance, that it was unconvinced that political science analysis is ready for a prime time role in detecting gerrymandering (LULAC v. Perry 2006; Grofman and King 2007). Also, while credible allegations of gerrymanders of congressional districts have been leveled against five states after the 2000 census (Mayhew 2011, 24; see also Toobin 2003) and against more than a dozen states after the 2010 census (Goedert 2014; see also Fang 2014), we are told that the allegations may be much ado about very little, because, "given the geographical concentration of Democrats in many areas, affirmative political engineering would be required to draw politically neutral congressional districts" (Bartels 2016, 48; see also Bishop 2008). The search for a standard for detecting gerrymanders continues. In 2015 and 2016 six different proposed methods for identifying gerrymanders were proposed in response to Common Cause's call for input on standards for identifying gerrymanders (McDonald and Best 2015; Chen and Rodden 2015; McGann et al. 2015; Tan and Liu nd.; Wang nd.; Arrington nd.).

hold sway over elections in those districts, but limit their influence to only that small number.

While both forms of gerrymandering can deliver one party more seats than they would win under a neutral process, the manipulation of outcomes occurs by very different processes and implicates different types of political harm to voters. Cracking gerrymanders induce a higher level of *responsiveness*, the rate by which votes are translated into seats, in a system of electoral districts. We say that systems subjected to cracking gerrymanders are overly responsive because in cracking gerrymanders a small change in a party's vote percentage results in a larger change in the number of seats a party should receive in a fair system. Cracking gerrymanders harm voters because in a polarized partisan environment where votes are unlikely to swing much in favor of one party and then the other, they deny one party's votes anything more than de minimis representation.

By contrast, packing gerrymanders induce *bias* in the way that votes are differentially weighted. By concentrating a party's voters in one or relatively few districts, an effective packing gerrymander ends up requiring that the disadvantaged party receive more than 50 percent of the votes to receive 50 percent of the seats. Such a violation of majority necessarily implies votes are differentially weighted inasmuch as violations of majority rule necessarily imply unequal vote weights (May 1952; Dahl 1989, 139-41). To be sure, packing gerrymanders can deliver excess seats to the advantaged party situationally; however, even in those situations the excess seats result from the differential weights the gerrymander places on votes cast for different parties. Thus, packing harms voters insofar as the gerrymander creates a context in which votes cast by one party's supporters count for less than the votes of the other party's supporters.

Because they are concerned with observing the number of excess seats a party wins, seat-focused metrics of gerrymanders complicate an analyst's task of distinguishing between cracking and packing's different effects. To avoid this problem, we propose a different approach that breaks with methods that focus on seats won. We show that it is possible to identify gerrymanders by observing their effects on the distribution of votes across electoral districts. Among the virtues of our approach, a focus on the vote distribution allows analysists to focus on first order (as opposed to second order) consequences of gerrymanders. An examination of the vote distribution allows an analysist to observe the direct, *first order* consequences of a gerrymander because the distribution of votes is readily observable and makes no demands for additional assumptions by an analyst. Characteristics of the seat-vote relationship are indirect, *second order* effects of a gerrymander because they require an analysist to make assumptions about how votes translate into seats.

Our proposed focus on the vote distribution allows an analyst to identify cracking and packing gerrymanders' different first order effects. Cracking gerrymanders have the first order effect of restricting the variance of vote distribution. A reduction in variance corresponds to a second order effect of a more responsive seat vote relationship. Packing gerrymanders have the first order effect of adding skew to the distribution of votes. A skewed distribution of votes corresponds to a second order effect of bias in the seat vote relationship.

This shifting of focus has at least four advantages.

(1) It permits separate analyses for cracking and packing gerrymanders. Considering the different effects of either type of gerrymander allows for important analytical, normative, and legal distinctions that a seat-focused approach can leave muddled.

(2) Analysis of the distribution of votes relies on the readily observable characteristics of the distribution. By contrast, seat-focused approaches require that an analyst impose a set of (unobservable) assumptions about how votes translate into seats.

(3) Characteristics of the distribution of votes across electoral districts are empirically and analytically manageable, requiring nothing more than a rudimentary understanding of statistics. Seat-focused approaches rely on unobservable models of vote-to-seat translation that require simulation methods, often computational intensive ones, and often specialized knowledge to interpret appropriately.

(4) Because the vote distribution effects are first-order consequences, it is easy to infer that the choice of where to place district lines is the cause of the harm that gerrymandering produces. Establishing a baseline against which to evaluate whether gerrymandering has caused more and fewer seats undeservedly for two major parties proves difficult.

We begin our diagnosis in section two by discussing how both conceptual and analytical complications arise when seats are the focus of a gerrymandering analysis. Section three provides a synopsis of what we know about the vote distribution consequences of gerrymandering and how we can use that knowledge to provide valid and reliable causal analyses. Section four demonstrates the virtue of the approach we propose and the potential pitfalls of a seat-focused approach by examining a possible cracking gerrymander of Massachusetts' congressional districts and a possible packing gerrymander of Ohio's congressional districts. While the vote distribution convincingly demonstrates that the Massachusetts districts are not a gerrymander, seat-focused approaches offer mixed (i.e., unreliable) readings. Ohio's districts are a gerrymander by the evidence provided by the vote distributions, but the seat focused analyses again give mixed readings. We conclude with reflections on the benefits of having a valid and reliable standard for identifying gerrymandering.

2. Analyzing Gerrymandering with a Seat Focus

Partisan gerrymandering is the "practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give a political party an unfair advantage by diluting the opposition's voting strength'" (Vieth v. Jubelirer, 2004, 271 n. 1, quoting Black's Law Dictionary 1999, 696). The key aspect of the definition is an unfair advantage caused by diluting the opposition's voting strength. Thus, while it is tempting to think a party's unfair advantage means it wins more and the opposition wins fewer seats than each deserves, succumbing to the seat-focus temptation is ill-fated. Seats are won above or below seemingly just desserts as a consequences of any number of forces other than vote dilution—e.g., incumbency, campaign resources, residential distribution of partisans, strongly favorable or unfavorable short-term forces. It is difficult to separate the influence of vote dilution brought about by the location of district boundaries from the numerous alternative causal forces that could influence a party's success in winning seats.

Equally important, analyses focused on seats treat the vote dilution as an offense to a singular representational right. Cracking gerrymanders diminish or eliminate the representation available to voters of a particular party, but a denial of an appropriate level of representation covers only half of how gerrymanders harm voters. The other half is a packing gerrymandering's power to dilute votes in a direct sense. Vote dilution that results from a packing gerrymander offends the democratic principle that everyone's vote counts the same.

Cracking gerrymanders spread one party's votes evenly across districts so that they constitute sizable but losing minorities in all or nearly all districts. From a seat-focus standpoint, cracking exaggerates seat-to-vote responsiveness. In doing so it dilutes representational rights because the votes of a sizable segment of voters produce no or vanishingly few voices represented in the halls of government. Packing gerrymanders are different. They exaggerate bias in the seat-to-vote translation by concentrating many partisans of one stripe in a small number of districts and by giving partisans of the other stripe strong but not overwhelming majorities in the larger number of remaining districts. The effect of packing for the advantaged party is to entrench itself in majority legislative status even when it does not win a majority of the vote. Such violations of majority rule occur only if votes do not count equally.

Explanations are needed to appreciate three important consequences of the observations just offered. In the following three sub-sections, we show, first, treating gerrymandering as a singular concept concerned only with representational rights blurs the distinction between the two types of vote dilution associated with packing and cracking. Second, seat-focused analyses of exaggerated responsiveness face a difficult task of establishing a standard—a non-gerrymandered baseline—against which an alleged gerrymander is to be compared. Third, on questions of exaggerated bias associated with packing gerrymanders, seat-effect measures of bias may prove unreliable because the level of seat-denominated bias often varies with the percentage of votes a party receives.

2a. Two Types of Dilution

It is well known that in a predominant two-party system with elections contested under single-member district plurality rules, the majority party tends to win a seat bonus. Each vote percentage point above 50 brings with it seat percentages beyond proportionate responsiveness; for instance, a party's 52 percent of the vote might win 56 percent of the seats. Figure 1 shows a seat-vote relationship of this kind. The light-grey curve traces a vote to seat translation of the often remarked upon "cube law" formulation. In the competitive vote range of 40 to 60 percent, a one point difference in the vote percentage yields approximately a three percentage point difference in seats.

[Figure 1 about here]

For the purpose of an example, imagine the cube law function is the translation that a fair district plan would produce. A purely cracking gerrymander would exaggerate the seat-vote responsiveness so that it takes a form something like the black curve. There the responsiveness within the competitive vote range yields an approximate ten percentage point seat response for each one percentage point difference in votes. The seat consequence of this exaggerated responsiveness has the expectation that with just above 55 percent of the vote the majority party wins all or nearly all the seats. This wholesale exclusion of 45 percent of the voters from any representation produces a result so super-majoritarian as to appear unfair.

The apparent unfairness is not a per se differential treatment of the two parties and their voters. A uniform vote swing of ten points in favor the heretofore minority party is expected to have it win 100 percent of the seats should it win 55 percent of the vote. In a polarized partisan environment without any reasonable prospect for a large swing, however, a minority party would find little solace in the equal treatment in principle, inasmuch as a practical matter representation of the voices of its voters are totally excluded in this election and, so long as the polarized voting persists, in many elections to come.

Unlike cracking, the effect of bias associated with packing does not exclude a group's candidates of choice from winning any or a mere de minimis number of seats. Quite to the contrary, packing grants a limited number of safe seats to one party but then spreads that party's voters as ineffective voting minorities among a large number of districts. Figure 2 depicts bias of this sort. Here, as in Figure 1, we designate the light-grey curve as a trace of the seat-vote mapping for a fair district plan. The black curve traces the translation under a biased plan. Notice that under some particular ranges of vote percentages, 0 to 40 and 60 to 100, the biased plan's translation looks eminently fair. However, in the range between 40 and 60 percent of the vote, responsiveness is weak and votes of 50 to 55 percent are unable to win a majority of the seats.

Unlike cracking gerrymanders, the harm of a packing gerrymander is not exclusion from representation; the disadvantage party is essentially guaranteed a set number of seats. Packing violates representational rights conditionally, whether too few seats are won relative to votes depends on the percentage of the vote a party receives. The persistent offense is vote dilution in its more direct form: individual votes count unequally depending on which party a person supports. We know this because an arrangement of rules and procedures that violate majority rule must assign different vote weights (May 1952; Dahl 1989, 139). Said differently, if one party wins a majority of seats with less than a majority of votes, it must also be true in that system that the opposing party can only win a majority with more than a simple majority of votes. If different parties win a majority of seats with different percentages of votes, then the system must weight votes cast for the parties differently.

[Figure 2 about here]

It is not as if analyses of gerrymandering have failed to appreciate the twofold, different effects. Having shown, in collaboration with Guillermo Owen, that in competitive electoral circumstances "optimal" gerrymandering is expected take a packing form, Bernie Grofman, in collaboration with others, advocated a partisan symmetry standard for the Court's consideration (Grofman and King 2007; King, Grofman, Gelman, and Katz 2005). The Supreme Court, itself, appreciates that cracking and packing produce different effects implicating different values, saying the Court's jurisprudence has shown a "preference for a level of parity between votes and representation sufficient to ensure that significant minority voices are heard and … that majorities are not consigned to minority status" (see Davis v Bandemer 1986, 125, fn. 9). Even so, an evaluation of the "level of parity" in the seat-vote relationship appears to have eclipsed the distinction between packing and cracking by placing particular emphasis, apparently sole emphasis, on "ensuring significant minority voices are heard." This is not enough, and it need not be.[3]

2b. How Responsive?

Evaluating whether a vote to seat translation is overly responsive as a consequence of gerrymandering requires, first, a description of how responsive the translation is and, next, a benchmark for how responsive it would be absent gerrymandering. Social scientific analyses describing responsiveness reach back to the nineteenth century. A century-plus later it has largely settled on one or another variation of a bivariate equation to describe the relationships in Figure 1 (above).[4] Discouragingly, establishing a benchmark against which an analysis can evaluate what an equation's result tells us about exaggerated responsiveness continues to bedevil analyses.

One possibility for checking responsiveness if the districts are not gerrymandered involves comparing the responsiveness in the system of interest against the responsiveness of similar systems. If the responsiveness is atypical compared to other jurisdictions or in comparison to the earlier elections in the same jurisdiction, something untoward might well be happening. This is an approach taken by Nicholas Goedert, for example, in his investigation of post-2012 gerrymandering of congressional districts (Goedert 2014; 2015). It is also among the proposals Samuel Wang advances (Wang 2016; N.D.). The frustrating

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3 Notice the parity referenced by the Court is to "votes and representation" (i.e., seats), an implicit signal that the focus is exclusively on equal representational rights whereas, instead, it is cracking gerrymanders that implicate representational rights while packing gerrymanders implicate equal voting rights.

4 The conventional model, as Richard Niemi and Simon Jackman call it (Niemi and Jackman 1991, 194), is a bivariate relationship with seats estimated as a function of votes, under one or another of various estimators: e.g., linear (Dahl 1956, 148), logit/bilogit (Theil, 1970; Tufte 1973; King and Browning 1987), exponential Linehan and Schrodt (1978), extended beta-binomial (Engstrom 2013), and probit (Goedert 2014).

fact is that the long train of developments when estimating responsiveness reveals that it varies from one system to another and one time period to another (see, e.g., March 1957-58; Tufte 1973; Gudgin and Taylor 1979; King and Browning 1987; Gelman and King 1991; Engstrom 2013). There is no assurance that comparisons of a system to other systems or to the same system at an earlier time is an appropriate baseline against which we might evaluate a potential gerrymander. Residential patterns differ, party competition differs, contestation patterns differ, and the influence of gerrymandering within a baseline set differs.

One way around the difficulty is to specify a responsiveness baseline theoretically. Eric McGhee does this by identifying a gerrymander on the basis of comparing the number of wasted votes for each of two major parties (McGhee 2014; see also Stephanopolous and McGhee 2015). The core idea, referred to as an *efficiency gap*, is that gerrymandering causes one party's votes to operate less efficiently than the other's by spreading them in losing causes among a large number of districts, by packing many of them in lopsided districts, or both. To calculate the efficiency gap one first tallies the difference between each party's votes in excess of 50%+1 and adds that tally to the difference between each party's votes for losing candidates.[5] The tally is standardized by expressing it as a percentage of total votes cast.[6] McGhee claims that an equal number of wasted provides an assurance a plan is not a gerrymander, and after allowance for a degree of expected variability (provided by comparison to other systems, at other times in the same system, or both) a gerrymander of congressional districts is evident when, for a state with eight of more congressional districts, the efficiency gap calculation indicates the advantaged party wins two more seats than a fair allocation (Stephanopolous and McGhee 2015, 831, 837). Presuming equal

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[5] McGhee defines surplus votes as those "in excess of the number needed to win" (McGhee 2014, 68), which makes it curious to calculated them as votes beyond 50% + 1. The definition implies counting one more than the votes received by the losing candidate, as calculated by Andrew Hacker (Hacker 1964, 55-7; see also Whitford v. Gill 2016, 150-2, Greisbach J. dissenting).

[6] In a formula, *wasted votes* % = $\{(W_A - W_B) / \Sigma v_i\}*100$, where W_A and W_B are the wasted (surplus votes plus votes in district losses) for parties A and B, and v_i is the total vote system-wide in election *i* (see McGhee 2014, 68).

turnout among districts, McGhee has shown that the efficiency gap's standard for fairness implies a responsiveness value of 2.0 (McGhee 2014, 79-80).[7]

If what the efficiency gap asserts were true it would be quite an insightful theoretical development. We would have a gerrymandering standard secured on theoretical grounds. Unfortunately, we have to doubt the validity of wasted votes as a per se indicator of gerrymandering, because votes are wasted for reasons other than gerrymandering. This much is certain from a simple thought experiment. In a single-district state—which cannot possibly be gerrymandered—responsiveness of 2.0 results only when the winner wins 75 percent of the vote, meaning winning the Delaware or Wyoming congressional district with 60 percent of the vote reveals a gerrymander. Marking a responsiveness value of other than 2.0 as an indicator of gerrymandering is equally dubious empirically. It implies that all the systems that correspond to something similar to the cube law do so as the result of gerrymandering. It also implies that SMD systems in U.S. cities with a high degree of racial residential segregation and responsiveness values close t0 1.0 for African American presence on city councils are the product of pro-African American gerrymanders. Furthermore, because the wasted vote proposition treats gerrymandering as a singular concept without distinguishing between its cracking and packing forms, it runs together a simultaneous check for exaggerated responsiveness with a check for exaggerated bias. The result is unreliability. Sometimes false negative readings seep in because, under certain circumstances, bias and responsiveness co-vary and compensate for one another.

A third possibility for identifying a cracking gerrymander using seat results is to check for an overly responsive system without a direct appeal to a responsiveness ratio. This can be accomplished by producing a large number of computer generated neutral maps and setting expected district wins as the standard against which to compare the number of districts actually won. If a neutral plan indicates a party is expected to win, say, 60 percent of the seats with 55 percent of the votes but it actually wins 75 percent of the districts with

[7] As applied to congressional districts the wasted vote formula in fn. 4 is a background consideration, since a congressional district gerrymander is to be defined by the just mentioned two-seat rule, which has to be calculated based on a responsiveness ratio of 2.0. In application to state legislative districts Stephanopolous and McGhee suggest using actual wasted vote percentage calculation and identifying a gerrymander as an outcome outside the bounds of ±8%.

55 percent of the vote, we would seem to have good reason to infer, unless refuted on other grounds, that mapmakers have gone out of their way to create an undeserved advantage for one party. A potential problem is that, much like the wasted vote approach, comparing expected to actual district wins treats gerrymandering as a singular concept because responsiveness and bias effects are allowed to operate in combination. The result, similar to the efficiency gap standard, is a reliability problem. Under some circumstances a districting plan appears to be a gerrymander, but under different vote circumstances the same districting plan appears not to be a gerrymander.

2c. How Biased?

It is common to represent the seat-vote relationship as a bivariate relationship. Among others, Richard Niemi and Simon Jackman suggest that the bivariate equation is the "conventional … standard" approach to evaluating the fairness of vote. One or another form of a bivariate equation for estimating the vote to seat translations (see fn. 3), conveniently provides a numerical statement of both responsiveness and bias. Bias is read as the difference between 50 and the expected seat percentage when a party wins 50 percent of the vote. We do not dispute that a bivariate representation of the seat-vote relationship is conventional and standard, nor that such a representation is convenient; however, this approach to reading bias poses problems.

Votes seldom split 50:50, and thus to evaluate bias at the unobserved point of a 50:50 vote split requires a projection. For that reason Edward Tufte rejected the description; instead, he defined bias as the difference between the vote percentage a party receives and 50 at the point it wins 50% of the seats—i.e., ($V\% - 50 | S\% = 50$).[8] He expressly chose this definition because the seat-denominated calculation is a dual-quantity—the multiplicative product of his vote-denominated definition of bias multiplied by responsiveness (Tufte, 1973, 542-3, fn. 4, see also Grofman 1983, 301-4)—that requires making a projection for a hypothetical situation when votes split 50:50. Moreover,

[8] See Tufte (1973, 542-3) for his graphic and verbal characterization of bias. See his tables 1 and 6 for his reporting on the amount of bias. But note, also, in his Table 2 he uses the statistical significance of each of various intercepts as a means of detecting the existence of bias.

projecting seats, however probabilistically plausible (Grofman and King 2007), is what gave pause to Justice Kennedy's reluctance to endorse a partisan (seat-vote) symmetry standard when deciding whether an unconstitutional gerrymander was operating (LULAC v. Perry 2006, 420, commenting that courts are "wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs").

In addition to relying on projections, the seat-denominated reading of bias needs to rule out causes of the bias other than gerrymandering. If nothing else, it is proper to ask how much existing residential patterns contribute to bias. In particular, it is well known and widely acknowledged that Republicans hold a natural advantage associated with their more efficient residential distribution (Erikson 1972; Veith v. Jubelirer 2004, 289-90; Chen and Rodden 2013). We are unaware of an analysis of bias using a bivariate equation that has drawn this distinction.

Combining rather than separating responsiveness and bias effects also creates problems for McGhee's efficiency gap. As it is intended to apply to congressional districts (Stephanopolous and McGhee 2015, 831, 837), a majoritarian seat-vote ratio of two to one is sufficient for equalizing wasted votes. For example, winning 60 percent of the seats (10 points above 50) in association with winning 55 percent of the votes (5 points above 50) means wasted votes are equalized and thus there is no gerrymander. To the contrary, however, a two-to-one majoritarian seat-vote correspondence can arise despite a substantial bias, despite a packing gerrymander being in place. This means allowing a two-to-one majoritarian ratio to be a sufficient condition for concluding there is no gerrymander is liable to produce a false negative reading. A 40-40-60-65-70 vote distribution offers one simple example. The five vote percentages are asymmetrically distributed around the mean of 55, but the efficiency gap shows an equal number of wasted votes. Thus, despite the claim that the efficiency gap necessarily equals zero in the absence of bias,[9] it does not really.

[9] See Stephanopoulos and McGhee (2015, 834 and passim).

Comparing seats won to expectations based on neutrally generated districting plans also runs into problems. A return to and reflection on the example in Figure 2 makes it easy to explain why. Bias has the effect of winning seats below, at, or above expectations, conditional on the percentage of votes a party receives. Because bias is the result of packing and because packing all but guarantees the disadvantaged party a number of safe seats, there can be vote percentage circumstances where the disadvantaged party appears to be winning seats at or above expectations. In Figure 2, Party A is clearly disadvantaged when its vote percentages are between 40 and 60, but at or below 40 percent, it wins seats at or above fairness expectations. Therefore, a match or mismatch between expected and observed number of districts carried is not a per se robust and structural feature of a districting plan. In the face of a packing gerrymander, the match or mismatch varies depending on the vote percentage won.[10]

3. Gerrymandering: Vote Distribution Characteristics

The vote to seat translations shown in figures 1 and 2 above are second order offspring of the first order vote distribution descriptions of electoral responsiveness and bias. They represent the cumulative density functions, or what John Nagle calls the rank/density function (Nagle 2015), of the more basic vote distributions. Panels A and B in Figure 3 reconvert the cumulative functions back to their first-order density functions.

[Figure 3 about here]

Panel A makes it visually evident that the seat-vote relationship representing the cube law (the grey curve) has a much larger standard deviation than the distribution that,

[10] For a real-world example of the problem, consider Chen and Rodden's attempt to indicate a gerrymander by counting President Bush's 2000 or John McCain's 2008 district wins across Florida, in their academic and trial-related work (Chen and Rodden 2013a; 2013b; 2014). As noticed and noted by both Darling (2013) and McCarty (McCarty 2013; 2014) in the Florida court proceedings, a match or mismatch between expected and observed number of districts carried is not a persistent, structural feature of a districting plan. Darling analyzed his 5,000 map null set for nine pre-2012 statewide Florida elections in addition to the McCain-Obama presidential contest. For the McCain-Obama contest he found, as did Chen and Rodden, the expected number of McCain wins under the 2012 lines was 14, whereas the enacted districting plan had McCain winning 17—a result observed in less than one percent of the null set plans. However, Darling's analysis of the nine other elections showed the actual versus expected wins either matched (three elections), differed by one in favor of Republicans (three elections), or differed by one or two in favor of Democrats (three elections)—see Darling (2013, 16).

within competitive range of 40 to 60, leads to translating a one unit vote percentage difference into an approximate ten percentage point seat gain (the black curve). The smaller the standard deviation, the higher the seat-vote responsiveness, because the smaller the standard deviation the greater the number of highly competitive districts. On the other hand, panel B makes it visually evident that the difference between the seat-vote relationship representing the cube law (again, the grey curve) and a biased translation associated with a packing gerrymander (the black curve) comes via comparison of the symmetry of one distribution to the skew of the other. While all of this has been visualized and known for some time, more than a century, nearly all analyses of the effects of gerrymanders in the past sixty years, have attempted to ascertain the existence of a gerrymander by their second order effect on seats won. Since the mid-twentieth century, analysts have neglected gerrymander's first order effects on the distribution of votes. Given that analysis of the vote distribution has been absent from the literature for some time, we offer a brief review to highlight the general principles and their connection to evaluating suspicions of gerrymandering.

3a. Responsiveness and the Standard Deviation

In the first social scientific analysis of vote to seat translations of which we are aware, Francis Edgeworth used a normal distribution model to account for the greater than one-to-one responsiveness of the single-member districts used in late nineteenth-century British elections (Edgeworth 1898). To ensure the applicability of his model, he first checked the normality assumption for symmetry.[11] Having found little reason to worry, he showed that for the distributions he was analyzing—i.e., an approximately normal distribution with a standard deviation of 10—four percent of the area under the curve resided within 1 vote percentage point of the mean.[12] In practical political terms, his result

[11] He wrote,"… there is a certain approximation to the normal form. The symmetry proper to the law of error is evidenced by the close correspondence between the arithmetic mean and median …" (Edgeworth 1898, 534). He also checked for turnout bias by comparing the district mean vote percentage to the system-wide vote percentage (Edgeworth 1898, 536-7).

[12] This is because one-tenth of a standard deviation ($1/\sigma$, where $\sigma = .10$) in a normal distribution covers four percent of the area, $\Phi(.10) = .03983$. The derivative of a cumulative normal distribution

implies that for a vote distribution with a normal form with a standard deviation of 10 and centered at 50, four percent of the constituencies have vote percentages in the 49 to 50 percent range, and another four percent of the constituencies have vote percentages in the 50 to 51 percent range. Therefore, as the normal distribution swings uniformly (or approximately so) back and forth across a 50:50 vote percentage split, a party can expect a one point vote swing in either party's favor to bring with it an additional four percent of the seats. Others have made the same or a similar point both more (see Kendall and Stuart 1950; Theil 1970; Gudgin and Taylor 1979, 20-30) and less technically (see Butler 1951; 1952; Niemi and Deegan 1978).

Edgeworth's observation allows us to make a loose but nonetheless solid generalization: the smaller the standard deviation of the vote distribution the greater the responsiveness.[13] A cracking gerrymander all but necessarily implies that the standard deviation is smaller than the standard deviation of a neutrally drawn plan. To understand why, consider the extreme case of cracking gerrymander in which a party receives 49 percent of the vote in every district. The party would win no seats and the standard deviation of such a distribution is zero. Now suppose that the system is subject to a uniform vote swing in which the party's vote share increases by 1.1 percent in every district. Because all districts are densely concentrated at 49 percent in one election and 50.1 percent in the next, the party goes from losing every district to winning every one. It is the dense concentration that gives rise to the exaggerated responsiveness. Put another way, when the standard deviation of the distribution of votes is smaller than reasonably expected due to residential patterns, a cracking gerrymander is indicated.[14]

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at its mean, $[1/\sigma(2\pi)^{1/2}]$, is approximately the area under the normal curve within $.01/\sigma$ unit, where σ is expressed as a proportion.

[13] One condition that could disrupt this generalization is a standard deviation of a symmetrical but leptokurtic distribution. A check for this possibility as it pertains to gerrymandering can be performed easily by looking for a set of packed districts for each party, which would fatten the tails of the distribution, increasing the variance but compressing the variance in the vicinity of the mean (see Theil 1970, 1218-9; Gudgin and Taylor 1979, 71-2).

[14] If the standard deviation is larger than expected, it would amount to an affirmative gerrymander, but such a result is likely to be deemed both politically fair and legally permissible in the sense it would apparently be designed to drive responsiveness toward proportional representation—"judicial interest should be at its lowest ebb when a State purports fairly to allocate political power to the

3b. Bias and Skew

Analysis of electoral bias began in earnest in the mid-twentieth century work of David Butler, with special motivation supplied by the re-emergence of the cube law proposition. Just over a decade after Edgeworth's paper was published, a London born and former Glaswegian MP (1890-1906), James Parker Smith, appeared before the Royal Commission on Systems of Elections of 1909 and offered a mathematical formulation of the vote-to-seat translation, made famous years later by a reference to it in the *Economist* (1950). Crediting advice from Percy A. MacMahon, former president of the London Mathematical Society, Parker Smith described what is now referred to as the cube law.[15] He told the Commission that when the ratio of party votes is A:B the ratio of party seats will be at least $A^3:B^3$ (cited and quoted in Kendall and Stuart 1950, 183-4). While interesting and sometimes accurate, the cube law takes no account of bias. At mid-century, the omission of bias from the cube law conjecture drew the attention of David Butler (Butler 1951; 1952).

Several years before the *Economist* brought attention to the cube law conjecture Butler had begun his dissection of electoral bias in British general elections. He investigated in particular the partisan effects of both malapportionment and gerrymandering (Butler 1947, 284). His analysis of the 1950 and 1951 general elections renewed and extended his detailed analysis with explicit reference to the unaccounted for bias in the cube law claim (Butler 1951; 1952). Perhaps naturally, to remain consistent with the cube law's seat-vote relationship, Butler expressed bias in terms of seats by transforming the vote distribution into a cumulative form and developing a simulation that assumes uniform partisan swing across districts (or at a minimum a stochastic process that leaves the shape of a distribution the same—see Butler 1951, 329-30, fn. 1). The Conservative party, he found, was disadvantaged by what he called a malapportionment effect but was advantaged to an even greater extent by gerrymandering effect—a more efficient distribution of its votes among districts—in particular being relatively less packed in lopsided districts than the Labour

parties in accordance with their voting strength and, within quite tolerable limits, succeeds in doing so" (Gaffney v. Cummings 1973, 754).

[15] For an account of the advice MacMahon gave to Parker Smith, see Garcia (2006).

vote. About the 1950 General Election Butler concluded "bias in the system can be clearly seen … [i]f the party votes had been equal … the Conservatives would have had 35 more seats than Labour" (Butler 1951, 330).

Butler chose to describe bias as seat-denominated while acknowledging bias could be easily described as vote denominated, as Edgeworth had, by comparing the vote distribution median to its mean (Butler 1952, 276 fn. 1). The median district is the one that marks a party's seats on or just over the cusp of democracy's critical majoritarian mark. The only reason the median district percentage would not equal the jurisdiction-wide mean percentage among districts is for the vote distribution to be skewed, which is to use statistical words to say that one party's votes are relatively more packed than the other's. What is more and more important, when a party can win the median district without winning a majority of the votes jurisdiction-wide, we know with certainty that one party's votes are worth less. Despite a few analysts having used the median-mean comparison in years since (e.g., Rydon 1957; Erikson 1972) and despite the fact that Tufte's definition of bias, ($V\% - 50|S\% = 50$), is functionally another way of referring to the median versus mean difference, Butler's choice of describing bias as seat-denominated has been by far the common choice for describing bias in scholarly work ever since.

For the purpose of analyzing packing gerrymanders this choice of a seat description turns out to be neither necessary nor wise. It requires a full-fledged mapping of votes onto seats and thus requires leveraging bias *and* responsiveness simultaneously. This is unnecessary because bias in the distribution of votes is readily observable and can be identified independent of responsiveness. It is unwise because co-variation between responsiveness and bias can have compensatory effects that disguise what is readily observable as a packing gerrymander.

To summarize briefly, to identify a packing gerrymander the analysis of the distribution of votes has several advantages. One advantage is that it grounds an analysis on an observed fact. One of two parties had to win a majority of seats. Likewise, the median percentage of votes the party received is readily observable as is the mean number of votes a party received. A second advantage is that the difference between the median and mean supplies an indication of the magnitude of the shortfall in adherence to majority rule. For instance, suppose a party carries a majority of districts while receiving just 45 percent of the vote system-wide. This implies that the disadvantaged party needs to win something on the order of 55 percent of the vote to win a majority of the seats. A third advantage comes from the fact that a median-mean difference is known to have been caused by where the district lines are drawn. When counting votes system-wide, all votes contribute equally; if the votes count differently after division into districts, the difference must have been caused by the placement of the lines.[16]

4. Applications: Massachusetts and Ohio

We have argued that evidence of gerrymandering is observable in the distribution of vote percentages a party receives among a set of electoral districts. In addition to the metrics we suggest, additional checks are needed. For example, an analyst must confirm that any observed bias is something more than a transient occurrence. This can be checked by looking for similar magnitudes of standard deviations and median-mean differences among a number of different elections. In addition, it is possible that the observed bias arises as a consequence of residential patterns. This can be checked by comparing the observed median-mean difference to median-mean differences among a large set of neutrally drawn maps. We demonstrate that the approach we suggest is as simple and easily manageable as it sounds through gerrymandering checks of Massachusetts' and Ohio's congressional districts.

No Republican won a congressional seat in Massachusetts between 1996 and 2016. Over that 20 years period, that makes Massachusetts the only state with three or more congressional districts where members of one party were unable to win even a single seat. Is this because Massachusetts has enacted a cracking gerrymander? Nicholas Goedert suspects that it could be, at least in the elections following the 2010 census (Goedert 2014,

[16] Before reaching that conclusion a check on turnout as a source of contra-majoritarian bias is needed. The check is easy; it requires comparing the system-wide two-party percentage to the mean two-party percentage. The system-wide percentage weights each voter equally; the mean weights each district equally. Any difference between them is the result of turnout differentials among districts, which needs to be discounted before assigning majority voter status to a party based on the mean district vote percentage.

4; 2015, 4). Turning to Ohio, 4 of its 16 districts elected Democrats in 2012, 2014, and 2016; these are the same four, and the only four, that Barack Obama carried in the 2012 presidential election when he won 51.5 percent of the major two-party vote. Is the restriction on Democratic victories to a mere one quarter of the districts won by Democrats even with a majority of the vote because Ohio has enacted a packing gerrymander? Some observers think it could be (Goedert 2014, 4; Fang 2014).

We re-evaluate both gerrymandering suspicions relying on the vote distribution characteristics, and we check them against 10,000 neutral maps for the degree to which residential patterns are responsible for any variance compression (cracking) or skew (packing). In addition, we apply the three seat-focused approaches reviewed above in order to evaluate how reliably they evaluate both suspicions.

4a. Data

Our interest lies with how the placement of district lines organizes partisan voters, asking whether the organization dilutes the representational or voting rights of one set of voters to the partisan advantage of the opposition. In 2012 and 2014, Democrats won all nine congressional districts in Massachusetts. In 2012 and 2014 congressional elections in Ohio, the median district Democratic vote percentage runs 7 to 8 percentage points below the mean district Democratic vote percentage. Given that these results refer to the congressional elections after competition has been influenced by where the district lines were drawn, we have to worry that they might result from forces other than gerrymandering: incumbency, quality of challengers, campaign contributions, campaign organization. In order to avoid potential problems of endogeneity, we focus on statewide races in which the nature of the candidate competition is similar in all districts.

From the Daily Kos, we have district-by-district compilations of statewide races for data disaggregated to United States Census designated Voter Tabulation Districts (VTDs)[17]

[17] VTDs roughly correspond to state designated voting precincts; however, the correspondence to actual voting precincts is not precise. In practice, states re-precinct more frequently than they redistrict. States share their precinct boundaries with the Census Bureau once every ten years, so the VTDs we use to develop our neutral maps are almost certainly out of date by the 2012 and 2014 elections, requiring us to rely on estimated vote totals by VTD. On the other hand, the Census Bureau does ensure that the population reported for VTDs is accurate.

and covering 15 Massachusetts and 17 Ohio statewide elections, 2004 through 2014. We use the Daily Kos report of post-2011 district-aggregated results to calculate standard deviations and skewness in the enacted district plan; and we use the VTD shape files provided by the United States Census Bureau to draw 10,000 computer generated (neutral) plans. For each of the 10,000 neutral plans, we aggregate the VTD-level Daily Kos political data to the computer-generated districts and calculate the standard deviation and skewness for the distribution of each map of the districts drawn by the computer. The pattern we observe in standard deviations and skewness that arise in the set of neutrally generated maps characterizes the patterns in skewness and standard deviations that would arise as consequence of residential patterns.

Since data provided by the Daily Kos and computer-generated maps are central to our analysis, it is worth pausing for a moment to discuss both. First, the Daily Kos provides election returns for state-wide elections in all fifty states. Steven Wolf of the Daily Kos provides versions of these data disaggregated to VTDs. Using county-level returns, Wolf assigns votes to VTDs according to votes cast in the VTD in the 2008 presidential election and the proportion of the county's population living in a VTD. The disaggregation of Democratic votes to VTDs can be characterized by the following equation.

$$d_i^t = \delta_i D^t,$$

where d_i^t is the estimated number of votes cast for a Democratic candidate in VTD i in election t, δ_i is the proportion of a county's votes cast in VTD i for the Barack Obama in 2008, and D^t is the county-level count of Democratic votes for election t. To understand how this equation works, suppose in 2008 a VTD cast 10 percent of a county's votes for Obama and 15 percent of a county's votes for McCain. If at the county level in 2012, 100 votes were cast for the Democratic candidate for Governor and 200 votes were cast for the Republican in the same election, the equation estimates the VTD vote total as 10 votes for the Democrat. Wolf estimates Republican votes by substituting the proportion of a county's 2008 McCain vote cast in VTD i for δ_i and the county-level count of Republican votes in election t for D^t. Applying the equation to the Republican returns, Wolf would estimate that the hypothetical VTD cast 30 votes for the Republican candidates.[18]

Second, we use a computer algorithm to characterize the distribution of possible neutral maps. To establish a meaningful counterfactual against which we may compare the observed outcome of the redistricting process, we draw 10,000 maps of each state's districts without applying any constraints besides the constitutional requirements of geographic contiguity and population parity. We rely on a method proposed by Daniel Magleby and Daniel Mosessson which leverages a class of graph partitioning algorithms developed by computer scientists (Magleby and Mosessson 2016). Graph partitioning algorithms assign related computational tasks to cores in a computer's processor (or nodes in a cluster or supercomputer) in a way that balances computational load across processors. In its application to drawing legislative maps, the algorithm assigns adjacent (contiguous) VTDs to legislative districts in a way that balances population across districts. The algorithm is highly efficient permitting us to draw a large number of maps. Moreover, the algorithm produces maps in a way that shows no indication of bias. We use the algorithm to draw 10,000 maps that contain the appropriate number of congressional districts for Massachusetts and Ohio, and constrain the algorithm to produce districts with roughly equal (+/- 1%) population.

4b. Findings: Massachusetts

Figure 4 offers two preliminary overviews of the congressional district situation in Massachusetts. The left panel shows the Democratic two-party vote percentage distribution in the 2012 U.S. Senate election between Democrat Elizabeth Warren and Republican Scott Brown (noting, this election can be taken as representative of all 15 inasmuch as its correlation with the others range from .949 to .999). Eight of nine districts have

[18] To arrive at these estimates, Wolf clearly assumes that VTDs will perform roughly as they did in the 2008 presidential election (Wolf 2014). Of course, estimating VTD performance relies on the additional assumption that VTD-level vote tallies for the 2008 election are accurate. Even if the assumptions hold true, the process of disaggregation introduces measurement error. To check the amount of error, we compared the Daily Kos data to official VTD returns in Wisconsin where the state provides official tallies of VTD-level votes. For every election covered by the data, we found the estimated returns provided by the Daily Kos were closely correlated with the official returns. Correlations of the Daily Kos numbers and official vote totals are in a range from 0.87 and 0.97.

percentages in the 45 to 60 range, possibly indicating a compressed variation consistent with a cracking gerrymander. Notice, however, one district—District 7 in the Boston area—is substantially packed with a large percentage of Democrats. The right panel traces the seat to vote correspondence for all 15 statewide offices contested between 2004 and 2013. There we see a highly responsive seat-vote relationship. Across the vote range from about 48 to 60, seat percentages change from just above 20 percent to just below 100 percent. Loosely stated, this is a seven point change in seats per each one point change in votes. Do the clustering of the Warren vote seen in the histogram and the highly responsive seat-vote correspondence seen in the election-by-election tracing indicate the Massachusetts congressional districts constitute a cracking gerrymander? The one-word answer is *no*.

[Figure 4 about here]

Columns 1 and 2 of Table 1 report the statewide Democrats vote percentage (#1) and the district-win percentage (#2) for each office.[19] The votes range from a high of 73.4 percent in Martha Coakley's 2006 election as Attorney General to a low of 48.7 percent when Coakley lost to Brown in the 2010 U.S. Senate election. Notice, too, in elections that Democrats win with more than 60 percent of the vote, their candidate carries all nine districts. Columns 3 and 4 report the observed (#3) and the expected (#4) standard deviations, where the expectations are from 10,000 computer generated 9-district plans. In the 6 most competitive elections (i.e., those when Democrats won between 48 and 56 percent of the vote) the standard deviations range from 9 and 11. These are smaller than the standard deviation underlying the cube law (responsiveness of 3 to 1), and in the vicinity of the responsiveness Edgeworth observed in the late nineteenth century British elections (responsiveness of about 4 to 1). By those relative comparisons, there is nothing particularly abnormal about what we observed in the most competitive Massachusetts elections. The most relevant comparisons, of course, are to the expected standard deviations in the neutral maps. Those comparisons show that in every election the observed standard deviation value is actually higher than expected. In comparison to standard deviations expected simply

[19] A mean percentage runs an average of 0.8 of a percentage point above the respective statewide percentage, indicating eight-tenths of a point turnout bias favoring Democrats. In no election does the turnout bias change the majority vote status as indicated by the mean percentage.

from the residential patterns of Massachusetts Democrats and Republicans, there is no indication of excessive compression of the variance of the distributions and, therefore, no indication of a cracking gerrymander.

[Table 1 about here]

For completeness, columns 5 and 6 check the observed (#5) and expected (#6) skew of the distributions, using median-mean differences. All differences, observed and expected, are negative, meaning the plans as drawn and the neutral maps operate with a bias adverse to Democrats. We know from Figure 4 that Massachusetts districts are skewed due to the concentration of Democrats in District 7. Do the actual districts exaggerate the standard deviation by packing Democrats in District 7 but then compress it (by cracking) in the other eight districts? No, calculating the standard deviation among eight districts not including District 7 and comparing it to neutral district standard deviations while removing the single most packed district show the standard deviation from the eight actual districts are about a half to a full point above expectations. So, it is not as if Massachusetts packed one district, thereby exaggerating the nine-district standard deviation, and then cracked the other eight.

What would the three seat-focused approaches to identifying a gerrymandering tell us? The short answer is that they produce mixed results, variable conclusions depending on the election to which an analysis is applied. From Geodert's analysis comparing the results in Massachusetts to expectations based on the nationwide bivariate seat-vote relationship, the Massachusetts districts are labelled a pro-Democratic gerrymander. If, instead, the comparison is to the expectations using the results from the neutral maps, we find that the Massachusetts district plan is slightly more responsive than the expectations from the neutral maps, though not statistically significantly more. Estimating the logit formulation using the 15 election results as the units of analysis yields

$$\text{Log}_e\,(S_D/100 - S\%_D) = -.33 + 7.74\ \text{Log}_e\,(V_D/(100 - V_D) + e_i,$$
$$(.59)\quad (1.07)$$

compared to the following when the expected (mean) values of district wins for each of the 15 elections from the 10,000 neutral maps are used,

$$\text{Log}_e\,(S_D/100 - S\%_D) = .38 + 6.83\ \text{Log}_e\,(V_D/(100 - V_D) + e_i.$$
$$(.53)\quad(0.96)$$

Here *Y* and *X* are, respectively, the log odds of the Democratic two-party seat and vote percentages, the intercepts are estimates of seat-denominated bias, the slopes are estimates of responsiveness in the 40-60 vote range, and the intercept and slope standard errors are in parentheses below each. Importantly, the two slope values are not statistically significantly different from one another, indicating there is no difference between the observed and expected responsiveness, or, in other words, there is no indication of a cracking gerrymander.[20] Oddly, however, neither estimation indicates any statistically significant bias; both intercept terms are only less than three-quarters the size of their respective standard errors. Even more oddly, the intercept for the expected value is positive, suggesting, if anything, an unreliable but slight bias favoring Democrats. We know from the median-mean comparisons, however, that bias consistently runs against Democrats. The problem is that, depending on the nature of the skew and its associated bias, a logit equation is not actually the correct specification. This is because skew resulting in a bi-modality in the vote distribution renders the logit formulation inapplicable (see Gudgin and Taylor 1979, 72-3, Nagle 2015, fn. 11). Therefore, the intercepts and slopes in both equations are biased, leaving us with doubt as to how to read the indication of either responsiveness or bias.

Table 2 reports the results for the two other seat-focused analyses, the efficiency gap and the comparison of observed versus expected district wins. The efficiency gap analysis leads to a full range of mixed results. Comparing, as the efficiency gap recommends for congressional districts, the observed number of wins (column #1) to the required efficiency gap expectation based on a responsiveness ratio of 2-to-1 (column #2), we find ten elections are not gerrymanders, four are pro-Democratic gerrymanders, and one is a pro-Republican gerrymander (see designations in column #3). All of this amounts to receiving no reliable diagnosis from observing the efficiency gap.

[20] The comparison relies on estimating the equations jointly with an interaction term. The .91 difference in slopes (7.74 – 6.83) has a standard error of 1.44; thus $t = .63$ ($p \approx .7$)

The observed versus expected wins comparisons fare better, but they also deliver mixed readings. If we set the boundaries for an indication of "fair" seat results using a 95 percent confidence interval generated by our 10,000 neutral maps, 13 of 15 elections appear not to be gerrymanders. The gubernatorial election in 2010 and the senatorial election in 2012, however, fail the test.[21]

[Table 2 about here]

The Massachusetts congressional districts are not a cracking gerrymander. Democrats are not concentrated in a restricted range of winning percentages and thus are not organized in a way that unnaturally spreads Republican voters among the nine districts, which, if it were true, would make it nearly impossible for them to win even one district. If anything, the decision to pack a large number of Democrats in District 7, under the right circumstances (not in evidence here), could actually work to the advantage of Republicans. It is equally apparent why a focus on seats as the indication of gerrymandering makes it difficult to arrive at a reliable reading of the situation. The three approaches evaluated here each produce mixed readings. It is unclear what a social scientist or court would conclude based in the seat-focused approaches, and whatever conclusion was drawn it would be subject to dispute by selecting one or another election as the proper focal point of the analysis.

4c. Findings: Ohio

Similar to how we initiated our analysis of Massachusetts, Figure 5 offers two preliminary overviews of the congressional district situation in Ohio. The left panel shows the Democratic two-party vote percentage distribution in the 2012 Presidential election between President Obama and Republican Mitt Romney (this election can be taken as representative of all 17 inasmuch as its correlation with the others range from .881 to .997). The distribution is substantially skewed, with Obama carrying four districts with greater than 60 percent of the vote, losing 11 districts with vote percentages in the range from 40 to

[21] We can save the proper inference by specifying the question as to whether Massachusetts is a pro-Democratic cracking gerrymander. Comparing wins answers 'no' to that question. Still, that does little to save this approach from the charge that it produces varied readings.

50, and losing one other in the 30 to 40 percent range. The right panel traces the seat to vote correspondence for all 17 statewide offices contested between 2006 and 2012. We see a nearly flat relationship between seats and votes over the 40 to 55 percent range and then a steep rise in districts carried when Democrats win 56 to 62 percent of the vote. Do the skew and the lack of responsiveness in the 40 to 55 percent range indicate a gerrymander? *Yes*, given a reliable measure.

[Figure 5 about here]

Table 3 reports the Democratic two-party seat and vote percentages (columns #1 and #2) along with observed and expected median-mean district vote percentage differences (columns #3 and #4) in Ohio's 17 statewide elections.[22] All median-mean differences are negative, indicating electoral bias runs consistently against Democrats. In the competitive elections, with the Democratic vote percentage range between 45 and 55, the bias is typically about four and a half points, suggesting it would likely take a Democratic vote percentage approaching 55 before Democrats could be expected to win a majority of the districts. Comparisons to the results of the 10,000 neutral maps demonstrate that the observed bias is not attributable to residential patterns. Ohio's voters are not residing in patterns that could be expected to bias the vote aggregation into districts that persistently favor one party. Six of the 17 elections show small expected biases running against Democrats but the remaining 11 elections have small expected biases running against Republicans. Indeed, the largest negative value of bias we observed among the 170,000 neutral maps is -2.04, which is a smaller bias disadvantaging Democrats than any of the 17 results from the actual elections. Ohio's congressional districts are a clear gerrymander: sizable electoral biases persistently run against Democrats. As for recognizable vote dilution, in the four elections Democrats won with 51.67 to 53.33 percent of the vote they failed to carry a majority of the districts.

[Table 3 about here]

[22] A mean percentage runs an average of 0.35 of a percentage point above the respective statewide percentage, indicating just over one-third of a point turnout bias favoring Democrats. In no election does the turnout bias change the majority vote status as indicated by the mean percentage (see Table note).

Similar to what we observe for Massachusetts, the three seat-focused approaches to identifying a gerrymandering reach mixed results. Using the national seat-vote relationship as a baseline for comparison, Goedert found a strong pro-Republican gerrymander in Ohio's 2012 congressional election but a small one-seat pro-Republican gerrymander effect in 2014. If, instead, the comparison is to the expectations using the results from the neutral maps, we find that the Ohio district plan is substantially more responsive than the expectations from the neutral maps, but, contrary to fact, the bias is about as expected. Estimating the logit formulation using the 17 election results yields

$$\text{Log}_e\,(S_D/100 - S\%_D) = -.48 + 4.63\ \text{Log}_e\,(V_D/(100 - V_D) + e_i$$
$$(.17)\quad(0.67)$$

compared to the following when the expected (mean) values of seats from the 10,000 neutral maps are used,

$$\text{Log}_e\,(S_D/100 - S\%_D) = -.58 + 13.83\ \text{Log}_e\,(V_D/(100 - V_D) + e_i$$
$$(.17)\quad(0.70)$$

where the elements reported are the same as referenced above for Massachusetts. The main issue in Ohio is the amount of bias, as estimated via the intercepts. The two intercept values—i.e., a value of -.38 for the observed seat-vote relationship versus -.48 for the relationship based on expectation derived from the 10,000 neutral maps—are not statistically significantly different from one another. That result indicates there is no bias introduced by gerrymandering as matter of choice, inasmuch as whatever bias exists is attributable to residential patterns. This is not true. We have already seen that a large amount of asymmetry has been introduced by choice, and it is an asymmetry that renders Democrats the minority party in the delegation unless they win a vote percentage approaching 55. What is actually happening in these estimations is a reflection of the fact that the logit specification is incapable of handling bias introduced by bimodality in the vote distribution, just as we saw in Massachusetts.

Table 4 reports results for the two other seat-focused analyses, the efficiency gap and the comparison of observed and expected district wins. Once again, the efficiency gap analysis leads to a full range of results. Four elections show a pro-Democratic gerrymander, the four they won with more than 55 percent of the vote. Four other elections indicate no gerrymander. The remaining nine elections, all those with Democratic vote percentages between 44 and 55 percent are deemed to be pro-Republican gerrymanders. Succinctly put, we do not receive reliable diagnoses from the efficiency gap approach.

[Table 4 about here]

Comparisons of observed versus expected wins perform no better; there we see mistaken readings 7 of 17 times. Observed versus expected wins in the 2008 election for Attorney General plus three elections in 2010 indicate no gerrymander, and when applied to the three elections when Democrats win less than 45 percent of the vote the comparisons actually indicate a pro-Democratic gerrymander. A modest preponderance of the evidence goes in the correct direction, but that does little to save the approach from the charge that it produces unreliable readings.

Ohio's congressional districts are a substantial pro-Republican gerrymander. Democratic votes are diluted; they are relegated to electing a minority of the congressional delegation even when they cast up to, perhaps, a vote percentage approaching 55. This gerrymander's majoritarian barrier requires Democratic voters to climb a steep hill just for the system to stay true to the basic democratic principle of majority rule. While the evidence supporting that conclusion is strong and convincing, attempts to diagnose Ohio's gerrymander by an appeal to seat-focused results tells us little upon which we would want to rely. The generalizable answer from those approaches is this. Whether Ohio's congressional districts constitute a gerrymander depends on how large the Democrat's vote percent is; sometimes the seat results appear to be fair, other times they do not. Fairness appears when Democrats win either a large or small percent of the votes; unfairness occurs for any outcome when Democrats threaten to or actually win a sizable but not overwhelming vote majority. Such conclusions amount to an invalid and unreliable way to diagnose gerrymandering.

5. Conclusion

Gerrymandering can distort the operation of democracy. At particular times and places it has, surely, but being able to diagnose when and where has proved difficult. The difficulty, as we explain, is a byproduct of two related analytical choices. Social science analyses have often treated gerrymandering as a singular concept and looked for gerrymandering effects by asking whether one party wins more and its opposition fewer seats than some definition of fairness would warrant. As realizable facts, however, gerrymandering is a dual concept with two effects that differ in kind. Cracking gerrymanders yield undeservedly large and small seat rewards for two major parties; packing gerrymanders often produce this effect only conditionally, depending on the level of the parties' vote percentages. When these differences in kind are not taken into account inferences and conclusions are often going to be unreliable. We can resolve the difficulties by approaching gerrymandering with cracking and packing qualifying adjectives expressly attached, by acknowledging explicitly the different kinds of effects, and by grounding an analysis on the first-order effects that gerrymanders have on the shape of the vote percentage distribution. Adherence to these conceptual and analytical distinctions makes it possible to distinguish, validly and reliably, between non-gerrymandered district plans and gerrymandered ones, as the illustrative applications to Massachusetts' and Ohio's post-2010 congressional districts demonstrate.

References

Arrington, Theodore. N.D. "A Practical Procedure for Detecting a Partisan Gerrymander." Election Law Journal.

Bartels, Larry M. 2016. "Elections in America." ANNALS of the American Academy of Political and Social Science 667: 36-49.

Bishop, Bill. 2008. The Big Sort: Why the Clustering of Like-minded America is Tearing Us Apart. New York: Houghton Mifflin.

Butler, David E. 1947. "The Relation of Seats to Votes." In R. B. McCallum and Alison Readman, The British General Elections of 1945. London, UK: Oxford University Press, 277-92.

Butler, David E. 1951. "Appendix." In H. G. Nichols, The British General Elections of 1950. London, UK: Oxford University Press, 306-33.

Butler, David E. 1952. The British General Elections of 1951. London, UK: Oxford University Press.

Cain, Bruce. 1985. "Excerpts from First Declaration of Bruce Cain in Badham v. Eu." PS 18: 561-7.

Cho, Wendy Tam and Yan Y. Liu. N.D. "Toward a Talismanic Redistricting Tool: A Fully Balanced Computational Method for Identifying Extreme Redistricting Plans. Election Law Journal.

Chen, Jowei and Jonathan Rodden. 2013a. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures." Quarterly Journal of Political Science 8: 239-69.

Chen, Jowei and Jonathan Rodden. 2013b. "Report on Computer Simulation of Florida Congressional Districting Plans" (February 15). Available at Justin Levitt's *All about Redistricting* website.

Chen, Jowei and Jonathan Rodden. 2014. "Response to Expert report of Stephen W. Hodge" (February 18). Available at Justin Levitt's *All about Redistricting* website.

Chen, Jowei and Jonathan Rodden. 2015. "Cutting through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders. Election Law Journal 14: 331-45.

Dahl, Robert A. 1956. A Preface to Democratic Theory. Chicago, IL: University of Chicago Press.

Dahl, Robert A. 1989. Democracy and Its Critics. New Haven, CT: Yale University Press.

Darling, Thomas A. 2013. "Report of Thomas A. Darling in Romo v. Detzner" (April 8). Available at Justin Levitt's *All about Redistricting* website.

Davis v. Bandemer. 1986. 478 U.S. 109.

Economist. 1950. "Electoral Facts." The Economist January 7: 5-7.

Edgeworth, Francis Y. 1898. "Miscellaneous Applications of the Calculus of Probabilities." Journal of the Royal Statistical Society 51: 534-44.

Engstrom, Erik J. 2013. Partisan Gerrymandering and the Construction of American Democracy. Ann Arbor, MI: University of Michigan Press.

Erikson, Robert S. 1972. "Malapportionment, Gerrymandering, and Party Fortunes in Congressional Elections." American Political Science Review 66: 1234-45.

Fang, Lee. 2015. "Gerrymandering Rigged the 2014 Elections for Republican Advantage." The Nation (November 5) www.the.nation.com/article/ gerrymandering-rigged-2014-elections-republican-advantage/

Gaffney v. Cummings. 1973. 412 U.S. 735.

Garcia, Paul. 2006. Life and Work of Major Percy Alexander MacMahon. PhD dissertation, Open University.

Gelman, Andrew and Gary King. 1991. "Estimating Incumbency Advantage without Bias." American Journal of Political Science 34: 1142-64.

Gelman, Andrew and Gary King. 1994a. "A Unified Method of Evaluating Electoral Systems and Redistricting Plans." American Journal of Political Science 38: 514-54.

Gelman, Andrew and Gary King. 1994b. "Enhancing Democracy through Legislative Redistricting." American Political Science Review 88: 541-59.

Goedert, Nicholas M. 2014. "Gerrymandering or Geography? How Democrats Won the Popular Vote but Lost the Congress in 2012." Research and Politics 1: 1-8.

Goedert, Nicholas M. 2015. "The Case of the Disappearing Bias: A 2014 Update to the 'Gerrymandering or Geography' Debate." Research and Politics 1: 1-6.

Griffith, Elmer C. 1907. The Rise and Development of the Gerrymander. Chicago: Scott, Foresman.

Grofman, Bernard. 1983. "Measures of Bias and Proportionality in Seats-Votes Relationships." Political Methodology 9: 295-327.

Grofman, Bernard. 1985. "Excerpts from First Declaration of Bernard Grofman in Badham v. Eu." PS 18: 544-50.

Grofman, Bernard and Gary King. 2007. "The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after *LULAC v. Perry*." Election Law Journal 6: 2-35.

Gudgin, Graham and Peter J. Taylor. 1979. Seat, Votes, and the Spatial Organisation of Elections. London, UK: Pion.

Hacker, Andrew. 1964. Congressional Districting: The Issue of Equal Representation (revised ed.). Washington, DC: Brookings.

Kendall, Maurice G, and Alan Stuart. 1950. "The Law of Cubic Proportion in Election Results." The British Journal of Sociology 1: 186-96.

King, Gary. 1989. "Representation through Legislative Redistricting: A Stochastic Model." American Journal of Political Science 33: 787-824.

King, Gary and Robert X. Browning. 1987. "Democratic Representation and Bias in Congressional Elections. American Political Science Review 81: 1251-73.

King, Gary and Andrew Gelman. 1991. "Systematic Consequences of Incumbency Advantage in U.S. House Elections." American Journal of Political Science 35: 110-38.

King, Gary, Bernard Grofman, Andrew Gelman, and Jonathan Katz. 2005. "Amicus Brief *in* Jackson v. Perry Submitted on Behalf of Neither Party, in the Supreme Court." (No. 05-276).

Linehan, William J. and Philip A. Schrodt 1977. "A New Test of the Cube Law." Political Methodology 4: 353-67.

LULAC v. Perry. 2006. 548 U.S. 399.

Magleby, Daniel B. and Daniel B. Mosesson. 2016. "Neutral Redistricting Using a Multi-Level Weighted Graph Partitioning Algorithm." Working paper.

March, James G. 1957-58. "Party Legislative Representation as a Function of Election Results." Public Opinion Quarterly 21: 521-42.

Martis, Kenneth C. 2008. "The Original Gerrymander." Political Geography 27: 833-9.

May, Kenneth O. 1952. "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision." Econometrica 20: 680-84

Mayhew, David 2011. Partisan Balance: Why Political Parties Don't Kill the U.S. Constitutional System. Princeton, NJ: Princeton University Press.

McCarty, Nolan. 2013. "Response to Chen and Rodden's 'Report on Computer Simulation of Florida Congressional Districting Plans'" (No date stated). Available at Justin Levitt's *All about Redistricting* website.

McCarty, Nolan. 2014. "Response to Jowei Chen and Jonathan Rodden's February, 2014 Report" (April 28). Available at Justin Levitt's *All about Redistricting* website.

McDonald, Michael D. and Robin E. Best. 2015. "Unfair Partisan Gerrymanders in Political and Law: A Diagnostic Applied to Six Cases." Election Law Journal 14: 312-30

McGann, Anthony, Charles Anthony Smith, Alex McKenna, and Michael Latner. 2015. "A Discernable and Manageable Standard for Partisan Gerrymandering." Election Law Journal 14: 295-311.

McGhee, Eric. 2014. "Measuring Partisan Bias in Single-Member District Electoral Systems." Legislative Studies Quarterly 55: 68–69.

Nagle, John F. 2015. "Measures of Partisan Bias for Legislating Fair Elections." Election Law Journal 14: 346-360.

Niemi, Richard D. and John Deegan, Jr. 1978. 'A Theory of Political Districting." American Political Science Review 72: 1304-23.

Niemi, Richard G. and Simon Jackman. 1991. "Bias and Responsiveness in State Legislative Redistricting." Legislative Studies Quarterly 16:183-202.

Owen, Guillermo and Bernard Grofman. 1988. Optimal Partisan Gerrymandering. Political Geography Quarterly 7: 5-22.

Rydon, Joan. 1957. "The Relationship of Votes to Seats in Elections for the Australian House of Representatives, 1949-1954." Political Science 9: 49-62.

Stephanopoulos, Nicholas and Eric McGhee. 2015. "Partisan Gerrymandering and the Efficiency Gap." University of Chicago Law Review 82: 831-900.

Theil, Henri. 1970. "The Cube Law Revisited." Journal of the American Statistical Association 65: 1213-19.

Thornburg v. Gingles. 1986. 478 U.S. 30.

Toobin, Jeffrey. 2003. "The Great Election Grab: When Does Gerrymandering Become a Threat to Democracy?" The New Yorker 79 (December 8): 63 ... 80.

Tufte, Edward R. 1973. "The Relationship between Seats and Votes in Two-Party Systems." American Political Science Review 67: 540-54.

Veith v. Jubelirer. 2004. 541 U. S. 267.

Wang, Samuel. 2016. "Three Tests for Practical Evaluation of Partisan Gerrymandering." Stanford Law Review 68: 1263-1321.

Wang, Samuel. N.D. "Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin." Election Law Journal.

Whitcomb v. Chavis. 1971. 403 U.S. 124.

White v. Regester. 1973. 412 U.S. 755.

Whitford v. Gill. 15-cv-421-bbc, 2016 U.S. Dist. LEXIS 160811 (W.D. Wisc. November 21, 2016).

Wolf, Stephen. "DRA Update: 2012 President & Downballot Election Results Estimates + Display Table Template." Daily Kos. 4 August 2014. Accessed 27 January 2017. <http://www.dailykos.com/stories/2014/8/4/1318876/-DRA-Update-2012-President-Downballot-Election-Results-Estimates-Display-Table-Template>.

Figure 1: Hypothetical Seat-Vote Relationship for a fair district map (grey curve) and a cracking gerrymander (black curve)

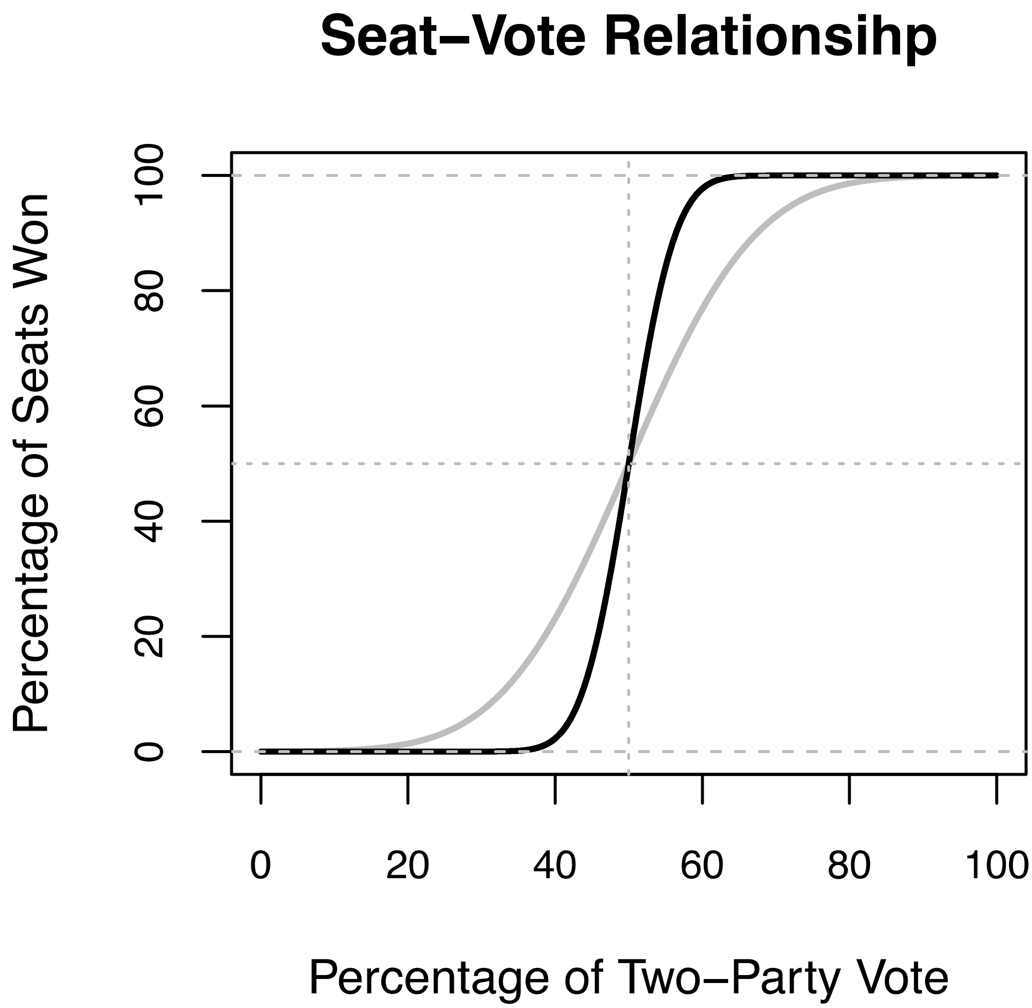


Figure 2: Hypothetical seat-vote relationship for fair district map (grey curve) and a packing gerrymander (black curve).

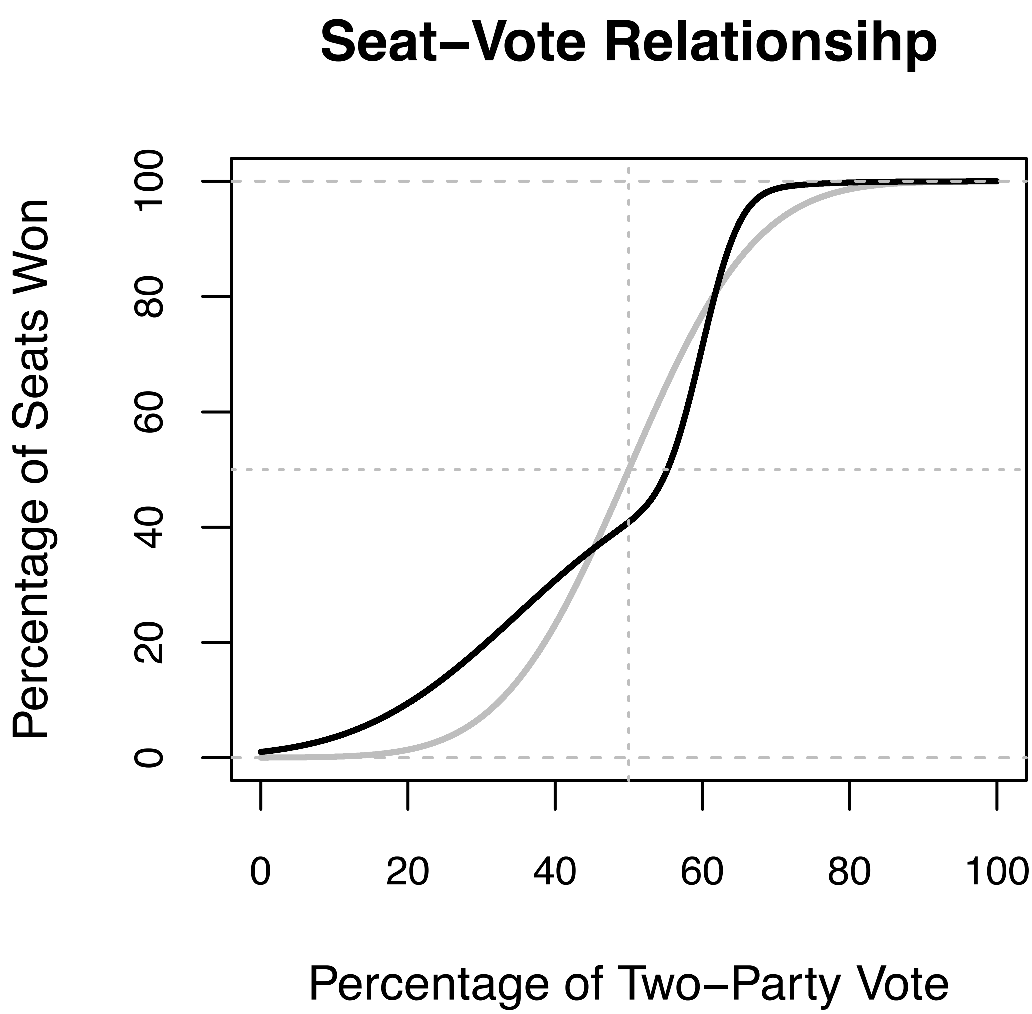


Figure 3: Hypothetical constituency vote percentage distributions for a cracking gerrymander (black curve in the left panel) and a packing gerrymander (black curve, right panel). Cracking gerrymanders reduce the variance in the vote distribution. Packing gerrymanders introduce skew into the distribution. For reference, we draw a hypothetical fair vote distribution (grey curve) corresponding to a normally distributed vote percentages.

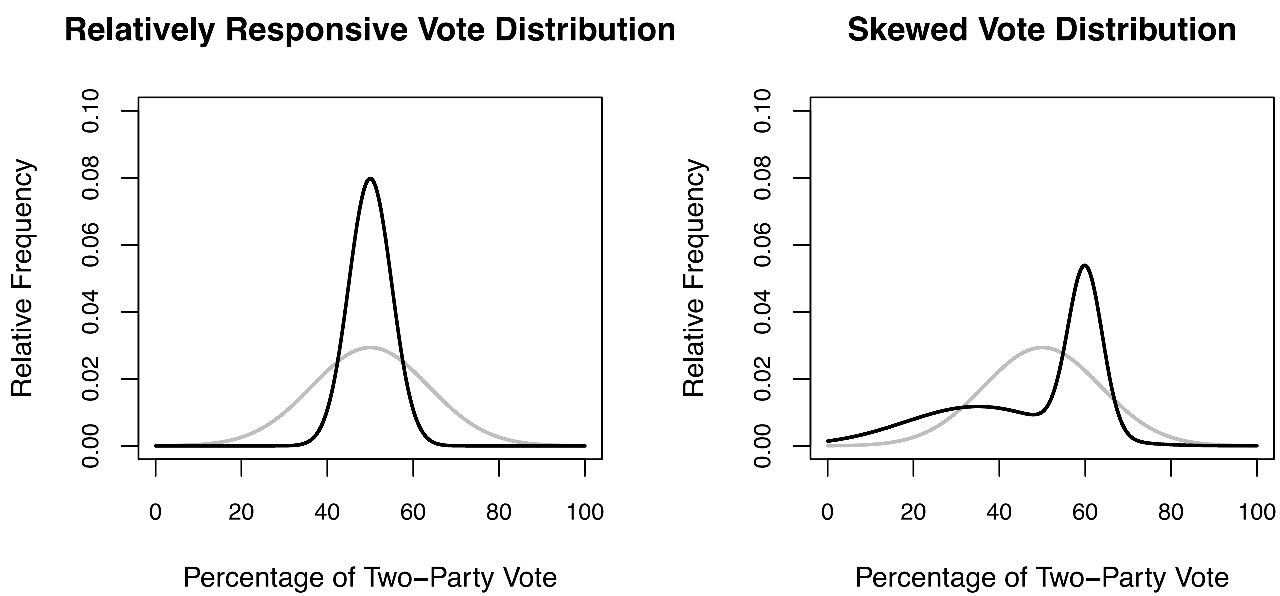


Figure 4: Vote Distribution in Massachusetts' 2012 U.S. Senate Election (left panel) and District Wins and Vote Relationship For Fifteen Statewide Elections, 2004-13

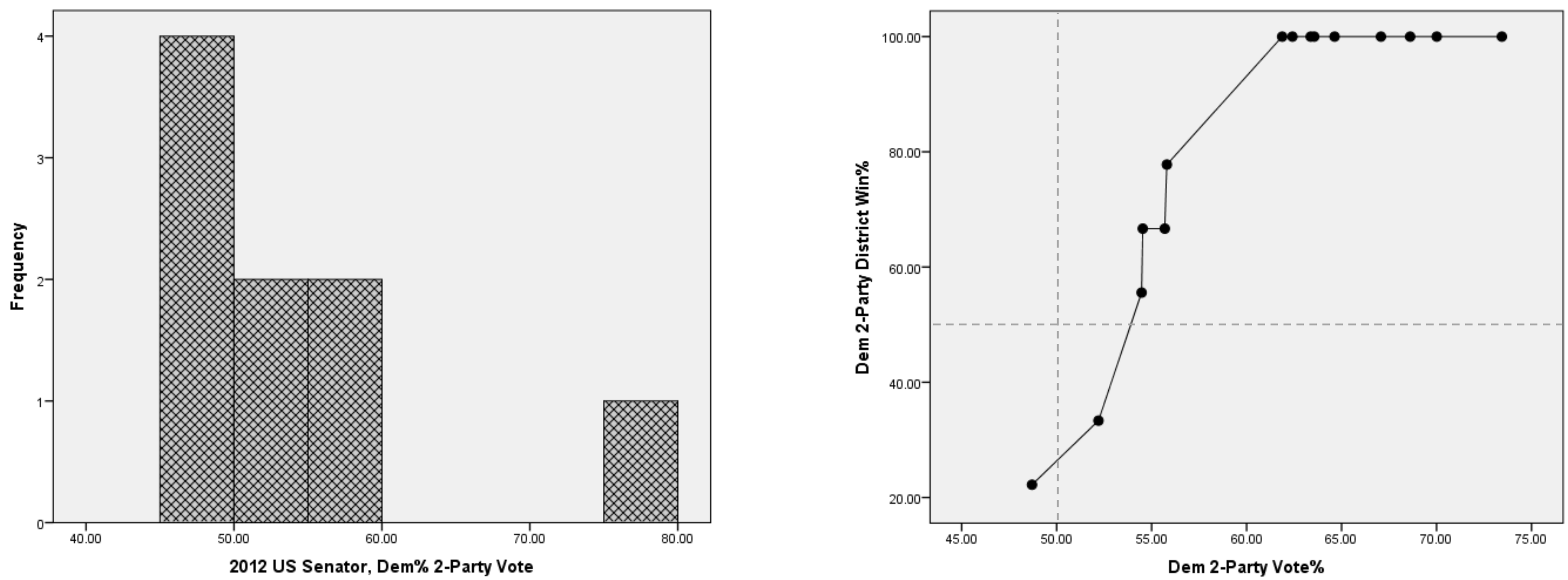
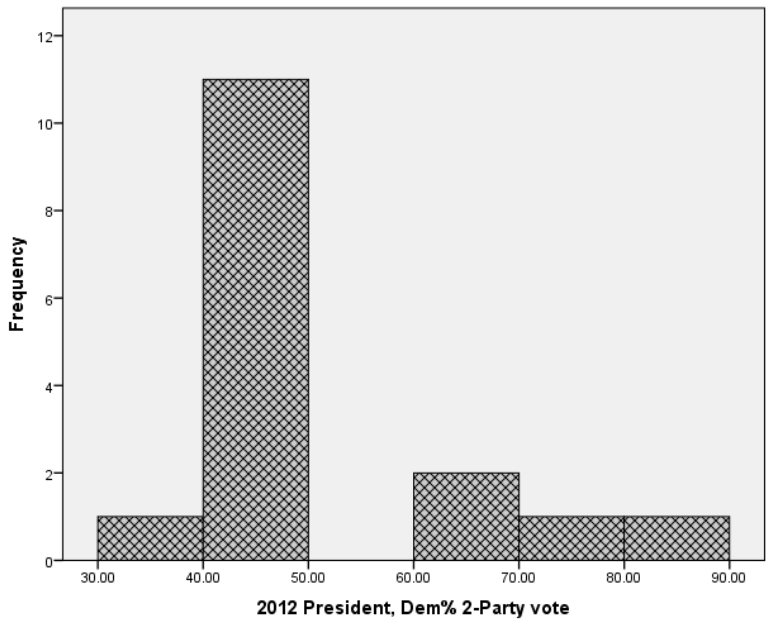


Figure 5: Vote Distribution in Ohio's 2012 Presidential Election (left panel) and District Wins and Vote Relationship For Seventeen Statewide Elections, 2004-12



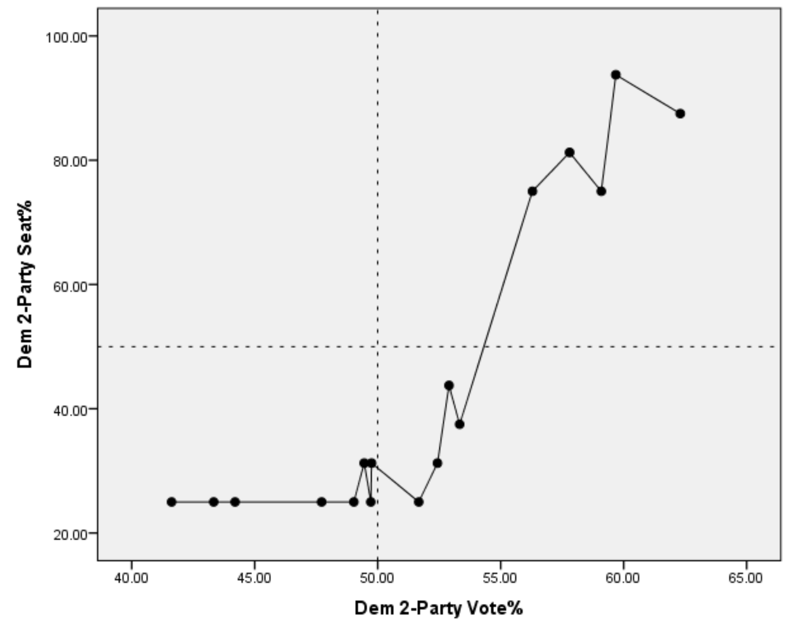


Table 1: Constituency Votes, Seat, Standard Deviations, and Median-Mean Difference in Fifteen Massachusetts Statewide Elections 2004-13

Office & Election	#1* Mean Vote %	#2 Seat %	#3 Observed Std Dev	#4 Expected Std Dev	#5 Observed Mdn–Mean	#6 Expected Mdn–Mean
Atty Gen 06	73.4	100.0	5.21	3.89	-1.58	-.72
US Senator 06	70.1	100.0	6.67	4.89	-1.60	-1.04
US Senator 08	68.6	100.0	7.30	5.36	-2.07	-1.42
Sec of State 10	67.1	100.0	7.93	5.83	-3.62	-1.41
President 08	64.6	100.0	10.51	7.74	-3.63	-2.28
Atty Gen 10	63.6	100.0	8.04	5.81	-3.68	-.99
President 04	63.4	100.0	7.45	5.35	-2.34	-1.24
President 12	62.4	100.0	8.92	6.54	-3.82	-1.82
Governor 06	61.9	100.0	7.97	5.28	-1.97	-1.59
Treasurer 10	55.8	77.8	9.89	7.03	-1.41	-1.08
US Senator 13	55.7	66.7	11.01	7.90	-4.67	-1.16
Governor 10	54.5	66.7	10.07	7.12	-4.13	-2.01
US Senator 12	54.5	55.6	9.91	7.26	-4.37	-2.11
Auditor 10	52.2	33.3	9.36	6.84	-3.13	-2.24
US Senator 10	48.7	22.2	10.52	7.41	-3.82	-1.62

*As noted in fn. 15, we need to be careful not to attribute majority vote status to a party without first checking for turnout bias by comparing the mean vote percentage to the statewide vote percentage. In Massachusetts, turnout bias favors Democrats by about one percentage point, but in none of the 15 elections does that change the mean indicator of majority vote status into a statewide minority vote. In sequence, top to bottom, the statewide vote percentages are 72.9, 69.4, 68.0, 66.3, 63.2, 62.8, 62.7, 61.7, 61.2, 55.1, 54.9, 53.8, 53.5, 51.1, and 47.6.

#1 Democratic candidate average vote percentage among nine districts.
#2 Percentage of nine districts carried by the Democratic candidate.
#3 Observed standard deviation of vote percentage among nine districts.
#4 Expected standard deviation of vote percentages among nine districts based on 10,000 maps drawn using a computer algorithm. (see Magleby and Mosesson N.D.).
#5 Observed difference between the median district vote percentage and the mean district vote percentage; negative values indicate a packing gerrymandering bias operating to the disadvantage of Democrats.
#6 Expected difference between the median district vote percentage and the mean district vote percentage, based on 10,000 maps drawn using a computer algorithm. (see Magleby and Mosesson N.D.). Negative values indicate a packing gerrymandering bias operating to the disadvantage of Democrats.

Table 2: Massachusetts Observed vs Expected District Wins under the Efficiency Gap and Neutral Map Standards for Detecting Gerrymanders

Office & Election	#1 Observed Wins	#2 Efficiency Gap Expected Wins	#3 Gerry-Mander?	#4 Neutral Expected Wins (95% CI)	#5 Gerry-Mander?
Atty Gen 06	9	8.7	No	9.0 (9,9)	No
US Senator 06	9	8.1	No	9.0 (9,9)	No
US Senator 08	9	7.8	No	9.0 (9,9)	No
Sec of State 10	9	7.6	No	9.0 (9,9)	No
President 08	9	7.1	No	9.0 (9,9)	No
Atty Gen 10	9	6.9	Pro-Dem	9.0 (9,9)	No
President 04	9	6.9	Pro-Dem	9.0 (9,9)	No
President 12	9	6.7	Pro-Dem	9.0 (9,9)	No
Governor 06	9	6.6	Pro-Dem	9.0 (9,9)	No
Treasurer 10	7	5.5	No	7.8 (7,9)	No
US Senator 13	6	5.5	No	7.5 (6,8)	No
Governor 10	6	5.3	No	7.5 (7,8)	Pro-Rep
US Senator 12	5	5.3	No	7.5 (7,8)	Pro-Rep
Auditor 10	3	4.9	No	4.4 (3,6)	No
US Senator 10	2	4.3	Pro-Rep	1.9 (1,3)	No

#1 Observed number of districts the statewide Democratic candidate carried.
#2 Expected number of districts carried by the Democratic candidate running statewide, based on the *efficiency gap* standard requiring equalized major-party wasted votes to yield a seat percentage difference from 50 be twice as large as a vote percentage difference from 50—i.e., implying (S% - 50) = 2(V% - 50) means two major parties waste an equal number of votes.
#3 The *efficiency gap* conclusion as to whether the elections indicates the district plan is a gerrymander favoring Democrats, Republicans, or neither, where the proposed decision rule is a difference between the observed and expected result is with $\pm$ 2 .
#4 Expected number of districts carried among nine districts based on 10,000 maps drawn using a computer algorithm. (see Magleby and Mosesson N.D.).
#5 The *neutral expected wins* conclusion as to whether the elections indicates the district plan is a gerrymander favoring Democrats, Republicans, or neither, where the observed outcome is outside a 95% confidence interval.

Table 3: Constituency Votes, Seat, Standard Deviations, and Median-Mean Difference in Seventeen Ohio Statewide Elections 2004-12

Office & Election	#1* Mean Vote %	#2 Seat %	#3 Observed Mdn–Mean	#4 Expected Mdn–Mean
Governor 06	62.3	87.5	-2.59	0.67
Atty General 08	59.7	93.8	-3.79	0.37
Treasurer 06	59.1	75.0	-4.21	0.56
Secof State 06	57.8	81.3	-4.86	0.14
US Senator 06	56.3	75.0	-4.52	0.04
US Senator 12	53.3	37.5	-4.58	-0.04
Atty General 06	52.9	43.8	-4.54	-0.15
President 08	52.4	31.3	-4.62	-0.06
President 12	51.7	25.0	-5.30	-0.01
Auditor 06	49.8	31.3	-4.19	0.39
Atty General 10	49.7	25.0	-4.66	0.18
Governor 10	49.5	31.3	-5.06	0.05
President 04	49.0	25.0	-4.12	0.16
Auditor 10	47.7	25.0	-3.82	-0.25
Sec of State 10	44.2	25.0	-5.10	0.05
Treasurer 10	43.3	25.0	-5.40	0.21
US Senator 10	41.6	25.0	-5.80	-0.03

*As noted in fn. 15, we need to be careful not to attribute majority vote status to a party without first checking for turnout bias by comparing the mean vote percentage to the statewide vote percentage. In Ohio, turnout bias favors Democrats typically by about one-third of a percentage point. In none of the 17 elections does the difference change the mean indicator of majority vote status into a statewide minority vote. In sequence, top to bottom, the statewide vote percentages are 62.3, 59.6, 57.9, 57.6, 56.2, 53.1, 52.6, 52.3, 51.5, 49.4, 49.3, 49.0, 48.9, 47.2, 43.6, 42.7, and 40.9.

#1 Democratic candidate average vote percentage among nine districts.
#2 Percentage of nine districts carried by the Democratic candidate.
#3 Observed difference between the median district vote percentage and the mean district vote percentage. Negative values indicate a packing gerrymandering bias operating to the disadvantage of Democrats.
#4 Expected difference between the median district vote percentage and the mean district vote percentage, based on 10,000 maps drawn using a computer algorithm (see Magleby and Mosesson N.D.). Negative values indicate a packing gerrymandering bias operating to the disadvantage of Democrats.

Table 4: Ohio Observed vs Expected District Wins under the Efficiency Gap and Neutral Map Standards for Detecting Gerrymanders

Office & Election	#1 Observed Wins	#2 Efficiency Gap Expected Wins	#3 Gerry-Mander?		#4 Neutral Expected Wins (95% CI)	#5 Gerry-Mander?
Governor 06	14	11.9	Pro-Dem		16.0 (16,16)	Pro-Rep
Atty General 08	15	11.1	Pro-Dem		15.9 (15,16)	No
Treasurer 06	12	10.9	No		15.3 (15,16)	Pro-Rep
Secof State 06	13	10.5	Pro-Dem		15.3 (15,16)	Pro-Rep
US Senator 06	12	10.0	Pro-Dem		15.1 (14,16)	Pro-Rep
US Senator 12	6	9.1	Pro-Rep		12.7 (11,14)	Pro-Rep
Atty General 06	7	8.9	No		11.6 (10,13)	Pro-Rep
President 08	5	8.8	Pro-Rep		11.9 (10,13)	Pro-Rep
President 12	4	8.5	Pro-Rep		10.4 (8,12)	Pro-Rep
Auditor 06	5	7.9	Pro-Rep		7.1 (5,9)	No
Atty General 10	4	7.9	Pro-Rep		6.9 (5,9)	Pro-Rep
Governor 10	5	7.8	Pro-Rep		6.3 (5,8)	No
President 04	4	7.7	Pro-Rep		6.5 (5,8)	Pro-Rep
Auditor 10	4	7.3	Pro-Rep		3.3 (2,5)	No
Sec of State 10	4	6.1	Pro-Rep		0.1 (0,1)	Pro-Dem
Treasurer 10	4	5.9	No		0.0 (0,0)	Pro-Dem
US Senator 10	4	5.3	No		0.0 (0,0)	Pro-Dem

#1 Observed number of districts statewide Democratic candidate carried.
#2 Expected number of districts carried by the Democratic candidate running statewide, based on the *efficiency gap* standard requiring equalized major-party wasted votes to yield a seat percentage difference from 50 be twice as large as a vote percentage difference from 50—i.e., implying (S% - 50) = 2(V% - 50) means two major parties waste an equal number of votes.
#3 The *efficiency gap* conclusion as to whether the election indicates the district plan is a gerrymander favoring Democrats, Republicans, or neither, where the proposed decision rule is a difference between the observed and expected result is within $\pm$ 2 .
#4 Expected number of districts carried among nine districts based on 10,000 maps drawn using a computer algorithm (see Magleby and Mosesson N.D.).
#5 The *neutral expected wins* conclusion as to whether the elections indicates the district plan is a gerrymander favoring Democrats, Republicans, or neither, where the observed outcome is outside a 95% confidence interval.